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Comparison of Methods for Determining the Physical Parameters of the Resonator of a Solid-State Wave Gyroscope

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Abstract. The article deals with the problem of identifying the dynamic parameters of the resonator of a solid-state wave gyroscope, based on the signals measured when the sensor is operating in free-run mode. The search for the dynamic parameters of a solid-state wave gyroscope is one of the most important operations of the quality control of its production. The paper describes two methods for determining the physical parameters of a quartz resonator of a solid-state wave gyroscope. For each method, the mathematical substantiation of the relationship between the dynamic behavior of the resonator and its physical parameters is given. On the basis of each of the techniques, an algorithmic support for the extraction of the physical parameters of the resonator of a solid-state wave gyroscope is presented. The research of the accuracy of calculating the visual parameters by the described methods on experimental data of a resonator with known parameters has been carried out. The results obtained show the practical applicability of the described methods. An example of using the methods described in the work is the identification and control of the dynamic parameters of a quartz hemispherical resonator of a solid-state wave gyroscope at the technological stage of "balancing".

Keywords: solid-state wave gyroscopes, dynamic parameters, conjugate gradient method, q-factor, different q-factor, different frequency, rigidity axes, viscosity axis

INTRODUCTION

Production cycle of hemispherical resonator gyroscope (HRG) includes a lot of technological operations, such as, operations of balancing control, different frequency, different Q-quality, calibration and others [1–3]. These operations control the most important parameters that affect the accuracy of the output signals of the HRG, are performed by measuring the dynamic characteristics of standing waves of a quartz resonator [4–9]. Since the latter characteristics turn out to be strongly related to the dynamic parameters of the TVG resonator, therefore, the research of the accuracy of the models for identifying the physical parameters of the resonator was chosen as the topic of this article.

THE PARAMETRIC STRUCTURE OF THE MEASURED SIGNALS

The wave processes observed by the measuring device have the following internal structure [1–3]:

$$\begin{aligned} C(t) &= A(t)\cos\theta(t)\cos(2\pi\omega t+\varphi) - B(t)\sin\theta(t)\sin(2\pi\omega t+\varphi), \\ S(t) &= A(t)\sin\theta(t)\cos(2\pi\omega t+\varphi) + B(t)\cos\theta(t)\sin(2\pi\omega t+\varphi), \end{aligned} \tag{1}$$

where $A(t)$ – amplitude of the main vibrations, $B(t)$ – amplitude of the quadrature oscillations, $\theta(t)$ – angular orientation of the wave, $\omega(t)$ – average oscillation frequency, φ – initial phase displacement.

In the general case, these functions depend not only on time t, but also on the physical parameters of the HRG resonator:

$$\begin{aligned} A(t) &= \tilde{A}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t), \\ B(t) &= \tilde{B}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t), \\ \theta(t) &= \tilde{\theta}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t). \end{aligned}$$

where η – damping factor, $\Delta\eta$ – delta of the damping factor, θ_η – axis of maximum viscosity, $\Delta\nu$ – value of different frequency, and θ_ν – axis of maximum stiffness.

The mathematical model of the rate of change of variables in the free-coast mode (without control action) is described by the equations [9–11]:

$$\begin{aligned} &\dot{A}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = -\eta A(t) - 2\Delta\eta A(t)\cos 2(\theta(t)-\vartheta_\eta) - \Delta\nu B(t)\sin 2(\theta(t)-\vartheta_\nu), \\ &\dot{B}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = -\eta B(t) + 2\Delta\eta B(t)\cos 2(\theta(t)-\vartheta_\eta) + \Delta\nu A(t)\sin 2(\theta(t)-\vartheta_\nu), \\ &\dot{\theta}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = \frac{(-\Delta\eta(\sin 2\theta(t)\cos 2\vartheta_\eta + \cos 2\theta(t)\sin 2\vartheta_\eta) - g_3)A(t)^2}{A(t)^2 - B(t)^2} + \\ &\quad + \frac{(-\Delta\eta(\sin 2\theta(t)\cos 2\vartheta_\eta + \cos 2\theta(t)\sin 2\vartheta_\eta) + g_3)B(t)^2}{A(t)^2 - B(t)^2} + \\ &\quad + \frac{2\Delta\nu(\cos 2\theta(t)\cos 2\vartheta_\nu + \sin 2\theta(t)\sin 2\vartheta_\nu)A(t)B(t)}{A(t)^2 - B(t)^2}, \end{aligned} \tag{2}$$

or

$$\begin{aligned} &\dot{A}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = -\eta A(t) - 2\Delta\eta_c A(t)\cos 2\theta - 2\Delta\eta_s A(t)\sin 2\theta - \\ &\quad - \Delta\nu_c B(t)\sin 2\theta + \Delta\nu_s B(t)\cos 2\theta, \\ &\dot{B}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = -\eta B(t) + 2\Delta\eta_c B(t)\cos 2\theta + 2\Delta\eta_s B(t)\sin 2\theta + \\ &\quad + \Delta\nu_c A(t)\sin 2\theta - \Delta\nu_s A(t)\cos 2\theta, \\ &\dot{\theta}(\eta, \Delta\eta, \vartheta_\eta, \Delta\nu, \vartheta_\nu, t) = \frac{(-\sin 2\theta(t)\Delta\eta_c + \cos 2\theta(t)\Delta\eta_s - g_3)A(t)^2}{A(t)^2 - B(t)^2} + \\ &\quad + \frac{(-\sin 2\theta(t)\Delta\eta_c + \cos 2\theta(t)\Delta\eta_s + g_3)B(t)^2}{A(t)^2 - B(t)^2} + \\ &\quad + \frac{2(\cos 2\theta(t)\Delta\nu_c + \sin 2\theta(t)\Delta\nu_s)A(t)B(t)}{A(t)^2 - B(t)^2}, \end{aligned} \tag{3}$$

where $\Delta\eta_c = \Delta\eta\cos 2\theta_\eta$, $\Delta\eta_s = \Delta\eta\sin 2\theta_\eta$, $\Delta\nu_c = \Delta\nu\cos 2\theta_\nu$, $\Delta\nu_s = \Delta\nu\sin 2\theta_\nu$.

CALCULATION OF THE PARAMETERS OF THE RESONATOR BY OBSERVING THE COAST

Substituting all derivatives in (3) with first-order difference schemes on the sampling interval T:

$$\begin{aligned}
\frac{A^{i+1}-A^{i}}{T} &= -\eta A^{i+1/2} - 2\Delta\eta_c A^{i+1/2}\cos 2\theta^{i+1/2} - 2\Delta\eta_s A^{i+1/2}\sin 2\theta^{i+1/2} - \\
&\quad - \Delta\nu_c B^{i+1/2}\sin 2\theta^{i+1/2} + \Delta\nu_s B^{i+1/2}\cos 2\theta^{i+1/2}, \\
\frac{B^{i+1}-B^{i}}{T} &= -\eta B^{i+1/2} + 2\Delta\eta_c B^{i+1/2}\cos 2\theta^{i+1/2} + 2\Delta\eta_s B^{i+1/2}\sin 2\theta^{i+1/2} + \\
&\quad + \Delta\nu_c A^{i+1/2}\sin 2\theta^{i+1/2} - \Delta\nu_s A^{i+1/2}\cos 2\theta^{i+1/2}, \\
\frac{\theta^{i+1}-\theta^{i}}{T} &= \frac{(-\sin(2\theta^{i+1/2})\Delta\eta_c + \cos(2\theta^{i+1/2})\Delta\eta_s - g_3)A^{i+1/2}\cdot A^{i+1/2}}{A^{i+1/2}A^{i+1/2} - B^{i+1/2}B^{i+1/2}} \\
&\quad + \frac{(-\sin(2\theta^{i+1/2})\Delta\eta_c + \cos(2\theta^{i+1/2})\Delta\eta_s + g_3)B^{i+1/2}\cdot B^{i+1/2}}{A^{i+1/2}A^{i+1/2} - B^{i+1/2}B^{i+1/2}} \\
&\quad + \frac{2(\cos(2\theta^{i+1/2})\Delta\nu_c + \sin(2\theta^{i+1/2})\Delta\nu_s)A^{i+1/2}B^{i+1/2}}{A^{i+1/2}A^{i+1/2} - B^{i+1/2}B^{i+1/2}}.
\end{aligned} \tag{4}$$

where the left and right sides of (4) are taken from the observed quantities {A, B, theta}.

After the left side of the equation system are collected in the vector dimension $\mathbf{h}(N, 1)$, right in the matrix $\mathbf{M}(N, 5)$ and the unknown vector is denoted $\mathbf{r} = [\eta, \Delta\eta_c, \Delta\eta_s, \Delta\nu_c, \Delta\nu_s]$, the system (4) can be written in matrix form [9]:

$$\mathbf{h} = \mathbf{M}\times\mathbf{r}. \tag{5}$$

Since the system of equations (5) is redundant, its solution is possible through minimization of the residual. This leads to the matrix equation:

$$\mathbf{M}^{\mathrm{T}}\times\mathbf{h} = \mathbf{M}^{\mathrm{T}}\times\mathbf{M}\times\mathbf{r} \Rightarrow \mathbf{r} = \left(\mathbf{M}^{\mathrm{T}}\times\mathbf{M}\right)^{-1}\times\mathbf{M}^{\mathrm{T}}\times\mathbf{h}. \tag{6}$$

To find the parameters of the HRG in coasting mode remains the problem of finding the wave variables.

NUMERICAL CALCULATION OF WAVE VARIABLES

In equation (1) the basic information functions are A(t), $B(t)$, $\theta(t)$, which is found by minimizing the functional errors:

$$F = \sum_i \begin{pmatrix} \left[-C_i + A_i\cos\theta_i\cos(\tau_i) - B_i\sin\theta_i\sin(\tau_i)\right]^2 + \\ +\left[-S_i + A_i\sin\theta_i\cos(\tau_i) + B_i\cos\theta_i\sin(\tau_i)\right]^2 \end{pmatrix} \to 0,$$
$$\tau_i = 2(\pi\omega)_i \cdot i\cdot n + \varphi_i, \quad i = 0\ldots N. \tag{7}$$

To numerically minimize it by the vector of parameters, the conjugate gradient method was chosen.

TRANSITION TO NEW VARIABLES (p, q, r).

The calculation of the variables [A, B, θ] can be simplified by introducing the following new variables [8]:

$$cc = \frac{2}{N}\sum_i C_i C_i = A^2\cos^2(\theta) + B^2\sin^2(\theta) = \frac{1}{2}(A^2+B^2) + \frac{1}{2}(A^2-B^2)\cos(2\theta),$$
$$cs = \frac{2}{N}\sum_i C_i S_i = (A^2-B^2)\sin(\theta)\cos(\theta) = \frac{1}{2}(A^2-B^2)\sin(2\theta), \quad (8)$$
$$ss = \frac{2}{N}\sum_i S_i S_i = A^2\sin^2(\theta) + B^2\cos^2(\theta) = \frac{1}{2}(A^2+B^2) - \frac{1}{2}(A^2-B^2)\cos(2\theta),$$

Next we proceed from basis [A, B, θ] to a new basis:

$$\begin{gathered} p = cc - ss = (A^2-B^2)\cos(2\theta), \\ q = 2cs = (A^2-B^2)\sin(2\theta), \\ r = cc + ss = A^2 + B^2. \end{gathered} \quad (9)$$

As a result, equations (3) will take the form:

$$\begin{gathered} \dot{p} = 2\eta p + 2g_3 q + 2\Delta\eta_c r - 4AB\Delta\nu_s, \\ \dot{q} = -2g_3 p + 2\eta q + 2\Delta\eta_s r + 4AB\Delta\nu_c, \\ \dot{r} = 2\Delta\eta_c p + 2\Delta\eta_s q + 2\eta r, \end{gathered} \quad (10)$$

where $AB = \mathrm{sign}(B)\cdot\mathrm{sqrt}(r^2 - p^2 - q^2)$, as well as g_3 the projection of rotation rate of the earth.

Further, the system of equations (10) performed substitution derivatives on time-difference scheme first order in analogy to (4). The resulting system of equations is solved by analogy with (5), (6). And the result of its solution will be a vector [η, $\Delta\eta_c$, $\Delta\eta_s$, $\Delta\nu_c$, $\Delta\nu_s$].

At the last phase of computations, the values of physical parameters are calculated using its values using the following formulas:

$$\begin{gathered} \vartheta_\eta = \frac{1}{4}\arctan\left(\frac{\Delta\eta_s}{\Delta\eta_c}\right), \ \vartheta_\nu = \frac{1}{4}\arctan\left(\frac{\Delta\nu_s}{\Delta\nu_c}\right), \\ Q = \frac{\pi\omega}{\eta}, \ \Delta Q = \frac{\pi\omega}{\sqrt{\Delta\eta_c{}^2 + \Delta\eta_s{}^2}}, \ \Delta\nu = \sqrt{\Delta\nu_c{}^2 + \Delta\nu_s{}^2}. \end{gathered} \quad (11)$$

where θ_η – axis of maximum viscosity, θ_ν – axis of maximum stiffness, Q – Q-factor, ΔQ – varied Q-factor, $\Delta\nu$ – different frequency.

COMPARISON OF CALCULATION RESULTS

HRG dynamic parameters were calculated by two methods according to formula (11). The math package Scilab was used for calculations.

In the first method, first solved (7), then from equation (4) is constructed in the matrix equation (5) and at the end, a system of equations (6).

In the second method, first phase variables are calculated by formulas (8), (9), then system (5) is constructed and the system of equations (6) is solved.

For an example of calculating the parameters of the HRG resonator, the data obtained experimentally at the stand for balancing are used. The initial data were obtained from the HRG resonator in the form of signals (*C*, *S*) (1), the measurement time was 60 sec, the sampling frequency of the ADC was 33333 Hz.

Table 1 shows the results of calculating the parameters of the resonator. The calculation of one value of a slowly varying variable was carried out for 1 period of oscillations.

Table 1. Result of HRG resonator calculation

Parameters	**Ideal value**	**Through the identification of A, B, θ (1st method)**	**By calculating p, q, r (2nd method)**
Q-factor	4210350.328	4196531.783	4193124.846
Varied Q-factor (%)	7	7.21	2.96
Axis of maximum viscosity, Degrees	88.9	89.0547	86.654
Different frequency, Hz	0.0003	0.0002734	0.0005286
Axis of maximum stiffness, Degrees	61.4	60.9435	59.9534

CONCLUSION

In this paper, mathematical models for determining the parameters of the resonator of a solid-state wave gyroscope are considered. Two computational methods for revealing the resonator parameters are analyzed. Each method has its own pros and cons. For example, the method based on the numerical minimization of the error functional by the least squares method has a higher accuracy than the method for calculating the values of *p*, *q*, *r*. Moreover, the second method has a lower computational complexity in comparison with the method of minimizing the error functional. It is also worth noting that the calculation of the parameters *p*, *q*, *r* should be performed over the full period of resonator oscillations.

These algorithms for identifying the parameters of the resonator of a solid-state wave gyroscope can be widely used in the production of gyroscopic devices and devices based on it. For example, in such technological operations as: balancing, tuning and control.

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