DOI: 10.22213/2658-3658-2018-6-10

Influence of a Non-Rigid Connection on the Scattering Properties of a Cylindrical Inclusion

K. E. Abbakumov, B. Ch. Ee

Dept. of Electroacoustic and Ultrasonic Technics,
Saint Petersburg Electrotechnical University LETI,
Saint-Petersburg, Russian Federation
E-mail: keabbakumov@etu.ru

Received: June 25, 2018

The incidence of a plane wave on a cylindrical inclusion located in a semi-infinite solid space with an inhomogeneous distribution of the rigidity at the boundary between inclusion-enclosing medium is considered. The cylindrical inclusion model is given. The solution is solved by the finite element method. Scattering indicatrices are given for different angles of incidence of a plane longitudinal wave. A significant effect of the coupling stiffness on the amplitude of the scattered field is shown.

Keywords: plane wave, cylindrical inclusion, finite element method, scattered field, scattered indicatrix.

INTRODUCTION

Nondestructive ultrasonic methods take one of the leading positions among other methods of nondestructive testing. Ultrasonic methods allow solving a wide range of problems and identification of defects in various, mainly solid objects, arising during operation or during production. Wide use of ultrasonic methods is also related with the fact that the fatigue and strength properties of the monitoring object itself influence the propagation of the ultrasonic wave.

It can be noted, that the most common types of defects are nonmetallic inclusions, which are formed due to the inevitable ingress of particles of decomposing refractories into the melt.

Because of the random nature of the process of generation and growth inhomogeneities in the metal and passing through several different process steps non-metallic inclusions can relate to the main metal scrap by means of different connection types [1, 2]. It is noted [3] that the processes of interaction of elastic waves with such a structurally complex interface between the nonmetallic inclusion and the metal can't be described using conventional boundary conditions that establish continuity of the stress tensor components and the displacement vector at the boundary. In [4–7] it was proposed to consider the usual boundary, when the elastic wave interacts with boundary, the components of the stress tensor remain continuous, and the components of the displacement vector can undergo a "discontinuity". The validity of this proposal based on the analytical relations. In [1], the existence of such boundary conditions

© Abbakumov K. E., Ee B. Ch., 2018

is proved, and it is also shown that the stress at the interface generally determined by the expression:

$$\sigma = K\Delta\mathbf{u},$$

where σ – stress tensor, $\Delta\mathbf{u}$ – vector of discontinuities in displacements, K – a positive definite symmetric matrix of dimension 3×3, known as the "boundary stiffness matrix", the elements which have the dimension N/m^3. Generally, these coefficients can be in complex form, which makes it possible to model new processes in the contact zone.

Such a representation allows to consider a special phenomena in the contact zone using various combinations of dynamic flexibility elements and / or damping elements, which is ensured by the corresponding values of the imaginary and real parts. So, in this case of dry mechanical contact of rough surfaces, the values of KN (normal component of contact stiffness) and KT (tangential component of contact stiffness) are assumed to be purely real. In case when these surfaces are separated by a layer of a viscous liquid, KN is taken to be complex, and KT is purely imaginary. It follows from boundary conditions in the linear "slip" approximation that the range of values of KN and KT in the general case from 0 (there is no transfer of the corresponding component of the displacement vector through the interface) to ∞ (the total transmission of the corresponding displacement vector component across the boundary section). The correspondence between the specific values of KN and KT and the structure of boundary can be established by the most suitable model [3].

FORMULATION OF THE PROBLEM

We consider the problem of normally incident plane waves diffraction on an infinite cylinder. We will consider a model of compact cylindrical inclusion of radius a, located in a ring layer of infinite small thickness (Figure 1).

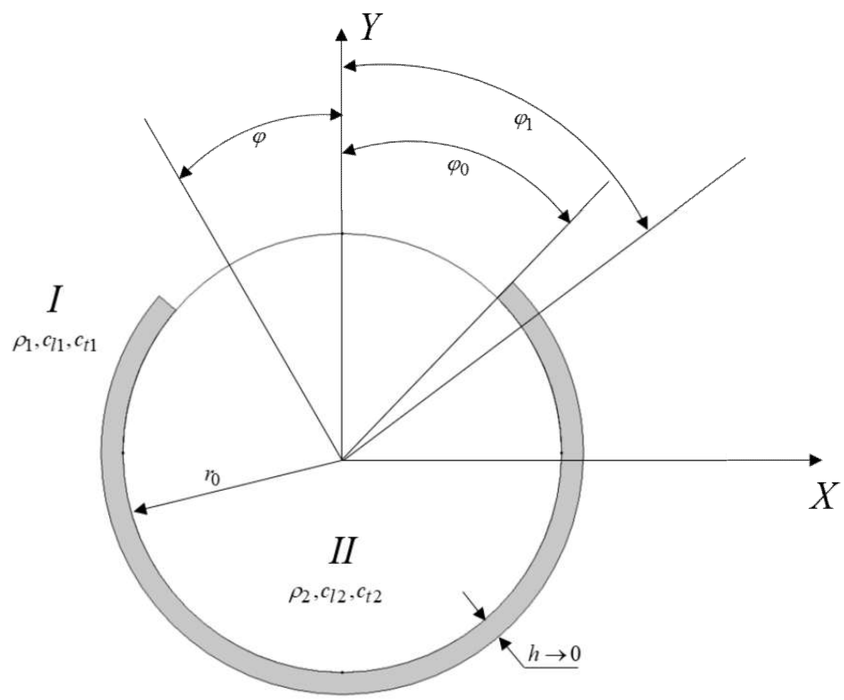

Figure 1. Formulation of problem

Let a plane wave incident on the cylinder (Figure 1) from the solid semi-infinite space I with physical parameters ρ_1, c_{l1}, c_{t1}. A rectangular coordinate system *XYZ* arranged so that the *Z* axis coincides with the longitudinal axis of the cylinder and the *Y* axis is directed along the bisector of the opening angle of the annular layer and the circular cylindrical coordinate system *r*, φ, *z* associated with the rectangular of known transformation formulas. The angle between wave vector and positive direction of the *Y* axis will be denoted by φ.

Solving the problem by the finite element method, a finite region was constructed (Figure 2), in which the cylindrical defect I, which is in the isotropic elastic space II. On the boundary between the defect and the elastic space in the sector of finite dimensions IV there is a nonrigid connection. The size of the sector was set as $\varphi_0 = L_{okr} dl$, where L_{okr} is the circumference in degrees, *dl* is a coefficient ranging from 0 to 1. It is of interest to analyze in infinite space a perfectly matched layer III is used, which absorbs all the waves entering into it.

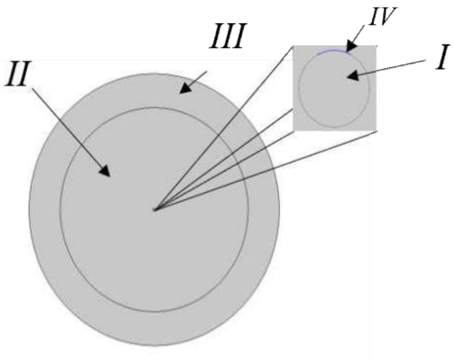

Figure 2. Investigated area

In entire investigated region, differential equation (1) is solving:

$$-\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \sigma. \tag{1}$$

In the stationary mode with harmonic perturbation, equation (1) can be rewritten in the following form:

$$-\rho \omega^2 \mathbf{u} = \nabla \sigma,$$

where ω – angular frequency, ∇ – Hamiltonian.

Setting the incident wave as an additional mechanical strength of the unit amplitude, equation (1) can be rewritten as

$$-\rho \omega^2 \mathbf{u} = \nabla \left(\sigma + \sigma_{ext} \right),$$

where σ_{ext} – stress in incident plane wave.

Boundary conditions for non-rigid sector are

$$\mathbf{u}_r^{II} = \mathbf{u}_r^{I} + \frac{\sigma_{rr}^{I}}{KGN(\varphi)}, \quad \sigma_{rr}^{II} = \sigma_{rr}^{I},$$

$$\mathbf{u}_\theta^{II} = \mathbf{u}_\theta^{I} + \frac{\sigma_{r\theta}^{I}}{KGT(\varphi)}, \quad \sigma_{r\theta}^{II} = \sigma_{r\theta}^{I}.$$

RESULTS

Figures 3–4 show the results of the calculation of a dimensionless displacement in polar coordinates, given as the ratio of the incident wave to the scattered one depending on the wavelength $k_a = \omega / c_l$, coefficient *dl* and angle of incidence φ. Figures 3*a* and 3*b* show that increase sector dimension with a non-rigid coupling, when the wave falls directly on the sector, the reflection in the opposite direction increases. Figures 4*a*–4*c* show the angular distribution of the dimensionless displacement, depending on the size of the sector. It is seen that for various angles of incidence, the scattering indicatrix is localized near the angle of incidence. It can also been seen that an increase sector dimension can lead both to an increase in the amplitude in the opposite direction (Figures 4*a*, 4*c*) and to a decrease in the amplitude (Figure 4*b*).

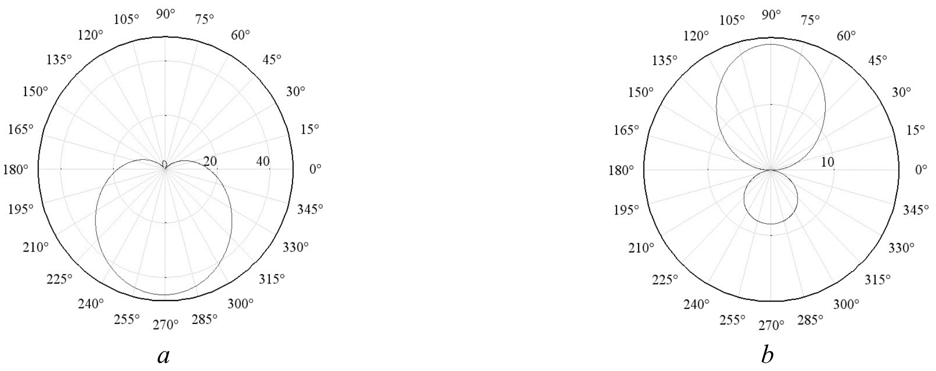

Figure 3. Dependence of the normalized amplitude of the scattered wave in polar coordinates at $k_a = 1.074$: $dl = 0$ (*a*); $dl_0 = 0.04$ (*b*)

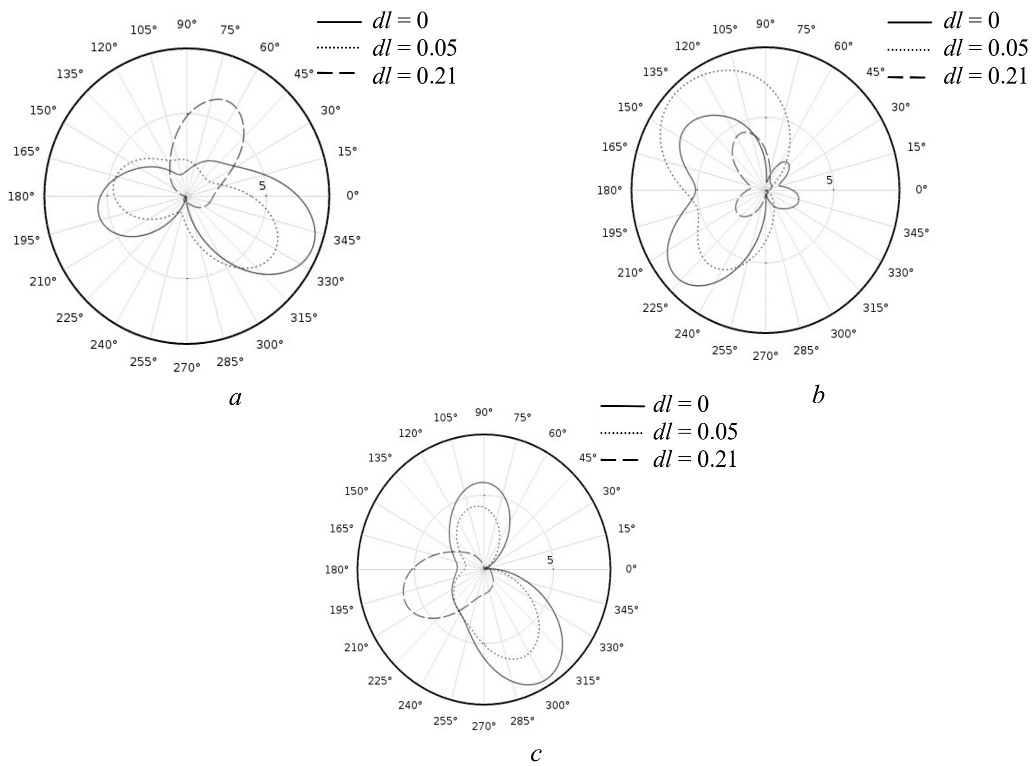

Figure 4. Dependence of the normalized amplitude of the scattered wave in polar coordinates at $k_a = 1.074$: φ = 24° (*a*); φ = 122° (*b*); φ = 196° (*c*)

CONCLUSION

The obtained results show a significant influence of non-rigid connection on scattering properties of a cylindrical inclusion. The size of the sector leads to increase the amplitude of the scattered wave in the opposite direction, if incident plane wave falls directly on sector with non-rigid connection. When incident plane wave falls to sector with rigid connection, the increasing in size of a sector with non-rigid connection leads to localizing scattered indicatrix near the falling angle. In practice, such influence can strongly affect the results of ultrasonic control. To obtain more accurate data about the internal structure, it's necessary to scan at different angles.

The results can be used to develop or modify methods for ultrasonic nondestructive control different objects with defects that can be approximate by cylinder.

REFERENCES

1. Margetan, F. J., Thompson, R. B., & Gray, T. A. (1988). Interfacial spring model for ultrasonic interactions with imperfect interfaces: Theory of oblique incidence and application to diffusion-bonded butt joints. *Journal of Nondestructive Evaluation, 7*(3–4), 131–152. doi: 10.1007/BF00565998.
2. Abbakumov, K. E., & Golubev, A. S. (1982). Issledovanie fizicheskikh modeley protyazhennykh neodnorodnostey v tverdykh telakh i ikh klassifikatsiya. [Study of physical models of extended inhomogeneities in solids and their classification]. In *Sb. tez. dokl. Vsesoyuz. nauch.-tekhn. konf. "Osnovnye napravleniya ul'trazvukovoy tekhnologii 1981–1990 gg." (15–17 dekabrya 1982) [Proceedings of All-Union sci.-tech. conference "Main directions of ultrasonic technology in 1981–1990" (December 15–17, 1982)]* (pp. 84–85), Suzdal', USSR : USSR Academy of Sciences (in Russian).
3. Maksimov, V. N. (1977). Prokhozhdenie akusticheskoy volny cherez tonkiy sloy mezhdu sherokhovatymi poverkhnostyami [Propagation of acoustic wave through thin layers between rough surfaces]. *Prikladnaya akustika [Applied Acoustics], 1977*(7), 132–135 (in Russian).
4. Huang, W., Rokhlin, S. I., & Wang, Y. J. (1997). Analysis of different boundary condition models for study of wave scattering from fiber-matrix interphases. *Journal of the Acoustical Society of America, 101*(4), 2031–2042. doi:10.1121/1.418135.
5. Schoenberg, M. (1980). Elastic wave behavior across linear slip interfaces. *The Journal of the Acoustical Society of America, 68*(5), 1516–1521. doi: 10.1121/1.385077.
6. Leiderman, R., Figueroa, J. C., Braga, A. M. B., & Rochinha, F. A. (2016). Scattering of ultrasonic guided waves by heterogeneous interfaces in elastic multi-layered structures. *Wave Motion, 63,* 68–82. doi: 10.1016/j.wavemoti.2016.01.006.
7. Lekesiz, H., Katsube, N., Rokhlin, S. I., & Seghic, R. R. (2013). Effective spring stiffness for a periodic array of interacting coplanar penny-shaped cracks at an interface between two dissimilar isotropic materials. *International Journal of Solids and Structures, 50*(18), 2817–2828. doi: 10.1016/j.ijsolstr.2013.04.006.